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Investigation of the behavior of a Griffith crack at the interface of a layer bonded to a half plane using the Schmidt method for the opening crack mode

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Abstract

In this paper, the behavior of a Griffith crack at the interface of a layer boned to a half plane subjected to a uniform tension is investigated by use of the Schmidt method under the assumptions that the effect of the crack surface overlapping very near the crack tips is negligible and also there is a sufficiently large component of mode-I loading so that the crack essentially remains open. By use of the Fourier transform, the problem can be solved with the help of two pairs of dual integral equations in which the unknown variables are the jumps of the displacements across the crack surfaces. To solve the dual integral equations, the jumps of the displacements across the crack surfaces are expanded in a series of Jacobi polynomials. Numerical examples are provided to show the effects of the crack length, the thickness of the material layer and the materials constants upon the stress intensity factor of the crack. As a special case in our solution, we also give the solution of the ordinary crack in homogeneous materials. Contrary to the previous solution of the interface crack problem, it is found that the stress singularities of the present interface crack solution are similar with ones for the ordinary crack in homogeneous materials.

Keywords: Elasticity; Interface crack; Schmidt method; Dual integral equations

1. Introduction

In recent years, composite materials and adhesive or bonded joints are being used in wide range of engineering field. The fracture of composites and bonded dissimilar materials is induced mainly from the interfacial region because the angular corner of bonded materials induces singular stress and crack initiation at the interface. Particularly flaws or cracks lying along the interface reduce the strength of the structure significantly. Hence, problem of interface cracks in dissimilar materials is very important from the

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view point of interface strength and stress analysis of interface cracks have been treated in many papers (England, 1965; Erdogan, 1965; Rice and Sih, 1965; Nisitani et al., 1993; Erdogan and Wu, 1993; Dhaliwal et al., 1989; Das and Debnath, 2001; Das and Patra, 1998; Erdogan and Gupta, 1971; He et al., 1992). A closed form solution of the interface crack problem was obtained in England (1965) by making suitable integral representation of the complex potentials developed by Muskhelishvili (1953). However, it is well known that the stress oscillatory singularity and overlapping of the crack surfaces appear near the interface crack tip and these are quite different from ordinary cracks in homogeneous materials. The solutions in Erdogan (1965), Rice and Sih (1965), Nisitani et al. (1993), Erdogan and Wu (1993), Dhaliwal et al. (1989), Das and Debnath (2001), Das and Patra (1998), Erdogan and Gupta (1971) and He et al. (1992) also contain the stress oscillating singularity as the same as in England (1965). Therefore, in comparison with the ordinary crack problems, it is difficult to analyse accurately the interface crack problem and there are not enough the data of stress intensity factors for interface cracks. In the papers Zhang (1989), Zhang (1986) and Itou (1986), the stress oscillatory singularity and overlapping of the crack surfaces do not appear near the interface crack tip for the opening interface crack mode. Some of the more significant results, particularly that concerning the discussion of the conditions leading to non-oscillating crack tip stress fields were given in Comninou (1977), Gautesen and Dundurs (1987), Atkinson (1982), Atkinson (1982) and Achenbach et al. (1979). However, the interface crack model was changed, i.e. the crack tips were assumed to be closed. In Achenbach's work (Achenbach et al., 1979), the interface crack problem was also studied. Non-oscillating crack tip stress fields were obtained in Achenbach et al. (1979). However, it was assumed that there was an adhesive zone at the crack tips.

Mathematically, the solutions in England (1965) and Erdogan (1965) are exactly forms in spite of the incomprehensibility in fracture mechanics. However, from an engineering viewpoint, it is more desirable to seek a solution which is physically acceptable (Itou, 1986). In the present paper, the same problem which was treated in Calhoun and Lowengrub (1978) is reworked by the use of a somewhat different approach, named as the Schmidt method (Yan, 1967). It is a simple and convenient method for solving this problem. The Fourier transform technique is applied and a mixed boundary value problem is reduced to two pairs of dual integral equations in which the unknown variables are the jumps of the displacements across the crack surfaces. To solve the dual integral equations, the jumps of the displacements across the crack surfaces are expanded in a series of Jacobi polynomials. This process is quite different from those adopted in England (1965), Erdogan (1965), Rice and Sih (1965), Nisitani et al. (1993), Erdogan and Wu (1993), Dhaliwal et al. (1989), Das and Debnath (2001), Das and Patra (1998), Erdogan and Gupta (1971), He et al. (1992), Comninou (1977), Gautesen and Dundurs (1987), Atkinson (1982), Atkinson (1982), Achenbach et al. (1979) and Calhoun and Lowengrub (1978) as mentioned above. During the solving process, the mathematical difficulties are not met, i.e. the oscillatory stress singularity and the overlapping of the crack surfaces do not meet. Contrary to the previous solution of the interface crack, it is found that the stress singularity of the present interface crack solution is of the same nature as that for the ordinary crack in homogeneous materials. As a special case, the solution of the present paper can be returned to the one of the ordinary crack in homogeneous materials.

2. Formulation of the problem

It is assumed that there is a crack of length $2l$ at the interface of a layer bonded to a half plane, h is the thickness of the layer. In terms of the rigidity modulus $G_j = \mu_j$, the Poisson's ratio η_j, the $k_j = \frac{\lambda_j + 3\mu_j}{\lambda_j + \mu_j}$ the Lame coefficients λ_j and μ_j where $j = 1, 2$ with 1 and 2 referring to the elastic layer and the lower half plane, respectively. In the case of plane strain the stress–strain relations may be written in the form (the results for plane stress may be derived easily by replacing the Poisson's ratio η_j, wherever it occurs by $\eta_j/(1 + \eta_j)$)

$$\sigma_x^{(j)} = \frac{2G_j}{1-2\eta_j}\left[(1-\eta_j)\frac{\partial u_j}{\partial x} + \frac{\partial v_j}{\partial y}\right], \quad (j=1,2) \tag{1}$$

$$\sigma_y^{(j)} = \frac{2G_j}{1-2\eta_j}\left[\eta_j\frac{\partial u_j}{\partial x} + (1-\eta_j)\frac{\partial v_j}{\partial y}\right], \quad (j=1,2) \tag{2}$$

$$\tau_{xy}^{(j)} = G_j\left(\frac{\partial u_j}{\partial y} + \frac{\partial v_j}{\partial x}\right), \quad (j=1,2) \tag{3}$$

where $\sigma_x^{(j)}$, $\sigma_y^{(j)}$ and $\tau_{xy}^{(j)}$ represent the Cartesian components of stress. u_j and v_j represent the Cartesian components of displacement. The subscript $j = 1,2$ with 1 and 2 referring to the elastic layer and the lower half plane throughout this paper, respectively. The governing differential equations can then be given as follows:

$$2(1-\eta_j)\frac{\partial^2 u_j}{\partial x^2} + (1-2\eta_j)\frac{\partial^2 u_j}{\partial y^2} + \frac{\partial^2 v_j}{\partial x \partial y} = 0, \quad (j=1,2) \tag{4}$$

$$(1-2\eta_j)\frac{\partial^2 v_j}{\partial x^2} + 2(1-\eta_j)\frac{\partial^2 v_j}{\partial y^2} + \frac{\partial^2 u_j}{\partial x \partial y} = 0, \quad (j=1,2) \tag{5}$$

The problem demonstrated in Fig. 1 will be solved under the following boundary conditions (in this paper, we just consider the perturbation field):

$$\sigma_y^{(1)} = \sigma_y^{(2)} = -\sigma_0, \qquad \tau_{xy}^{(1)} = \tau_{xy}^{(2)} = 0, \quad |x| \leqslant l, \ \ y=0 \tag{6}$$

$$v_1 = v_2; u_1 = u_2, \qquad \sigma_y^{(1)} = \sigma_y^{(2)}, \quad \tau_{xy}^{(1)} = \tau_{xy}^{(2)}, |x| > l, \ \ y=0 \tag{7}$$

$$\sigma_y^{(1)} = 0, \qquad \tau_{xy}^{(1)} = 0, \quad |x| < \infty, \quad y = h \tag{8}$$

$$u_j = v_j = 0, \qquad \sigma_y^{(j)} = \tau_{xy}^{(j)} = 0 \text{ for } \sqrt{x^2+y^2} \to \infty, \quad (j=1,2) \tag{9}$$

where the σ_0 is a magnitude of the uniform tension.

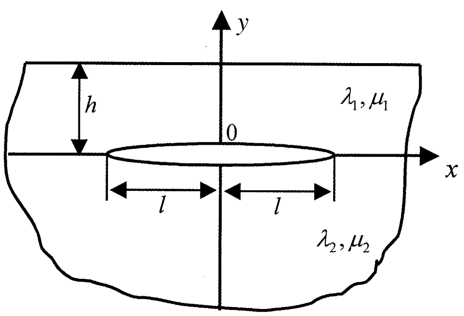

Fig. 1. Geometry and coordinate system for an interface crack.

3. Solution

Because of the symmetry, it suffices to consider the problem for $x > 0,\ |y| < \infty$. Eqs. (4) and (5) can be solved giving

$$u_1(x,y) = \frac{2}{\pi}\int_0^\infty s^{-1}[\{A_1(s) - k_1^{-1}[A_1(s) - B_1(s)]sy\}e^{-sy} + \{A_2(s) + k_1^{-1}[A_2(s) + B_2(s)]sy\}e^{sy}]\sin(sx)\,ds \tag{10}$$

$$v_1(x,y) = \frac{2}{\pi}\int_0^\infty s^{-1}[\{B_1(s) - k_1^{-1}[A_1(s) - B_1(s)]sy\}e^{-sy} + \{B_2(s) - k_1^{-1}[A_2(s) + B_2(s)]sy\}e^{sy}]\cos(sx)\,ds \tag{11}$$

$$u_2(x,y) = \frac{2}{\pi}\int_0^\infty s^{-1}\{A_3(s) + k_2^{-1}[A_3(s) + B_3(s)]sy\}e^{sy}\sin(sx)\,ds \tag{12}$$

$$v_2(x,y) = \frac{2}{\pi}\int_0^\infty s^{-1}\{B_3(s) - k_2^{-1}[A_3(s) + B_3(s)]sy\}e^{sy}\cos(sx)\,ds \tag{13}$$

where $A_1(s)$, $B_1(s)$, $A_2(s)$, $B_2(s)$, $A_3(s)$ and $B_3(s)$ are unknown functions to be determined by the boundary conditions. Substituting Eqs. (10)–(13) into (1)–(3), it can be obtained

$$\begin{aligned}\sigma_y^{(1)}(x,y) = \frac{-2k_1^{-1}G_1}{\pi}\int_0^\infty &[\{(k_1-1)A_1(s) + (k_1+1)B_1(s) - 2[A_1(s) - B_1(s)]sy\}e^{-sy} \\ &+ \{(k_1-1)A_2(s) - (k_1+1)B_2(s) + 2[A_2(s) + B_2(s)]sy\}e^{sy}]\cos(sx)\,ds\end{aligned} \tag{14}$$

$$\begin{aligned}\tau_{xy}^{(1)}(x,y) = \frac{-2k_1^{-1}G_1}{\pi}\int_0^\infty &[\{(k_1+1)A_1(s) + (k_1-1)B_1(s) - 2[A_1(s) - B_1(s)]sy\}e^{-sy} \\ &- \{(k_1+1)A_2(s) + (1-k_1)B_2(s) + 2[A_2(s) + B_2(s)]sy\}e^{sy}]\sin(sx)\,ds\end{aligned} \tag{15}$$

$$\sigma_y^{(2)}(x,y) = \frac{-2k_2^{-1}G_2}{\pi}\int_0^\infty [\{(k_2-1)A_3(s) - (k_2+1)B_3(s) + 2[A_3(s) + B_3(s)]sy\}e^{sy}]\cos(sx)\,ds \tag{16}$$

$$\tau_{xy}^{(2)}(x,y) = \frac{-2K_2^{-1}G_2}{\pi}\int_0^\infty \{(k_2+1)A_3(s) + (1-k_2)B_3(s) + 2[A_3(s) + B_3(s)]sy\}e^{sy}\sin(sx)\,ds \tag{17}$$

From Eqs. (6)–(8), we see that $\sigma_y^{(1)}(x,h) = 0$, $\tau_{xy}^{(1)}(x,h) = 0$, $\sigma_y^{(1)}(x,0) = \sigma_y^{(2)}(x,0)$, and $\tau_{xy}^{(1)}(x,0) = \tau_{xy}^{(2)}(x,0)$ for all values of x and it is easily shown that this condition is equivalent to equations

$$\begin{aligned}&\{(k_1-1)A_1(s) + (k_1+1)B_1(s) - 2[A_1(s) - B_1(s)]sh\}e^{-sh} + \{(k_1-1)A_2(s) - (k_1+1)B_2(s) \\ &\quad + 2[A_2(s) + B_2(s)]sh\}e^{sh} = 0\end{aligned} \tag{18}$$

$$\begin{aligned}&\{(k_1+1)A_1(s) + (k_1-1)B_1(s) - 2[A_1(s) - B_1(s)]sh\}e^{-sh} - \{(k_1+1)A_2(s) + (1-k_1)B_2(s) \\ &\quad + 2[A_2(s) + B_2(s)]sh\}e^{sh} = 0\end{aligned} \tag{19}$$

$$\begin{aligned}&k_2[(k_1-1)A_1(s) + (k_1+1)B_1(s) + (k_1-1)A_2(s) - (k_1+1)B_2(s)] \\ &\quad = k_1L[(k_2-1)A_3(s) - (k_2+1)B_3(s)]\end{aligned} \tag{20}$$

$$-k_2[(k_1+1)A_1(s)+(k_1-1)B_1(s)-(k_1+1)A_2(s)-(1-k_1)B_2(s)] \\ = k_1L[(k_2+1)A_3(s)+(1-k_2)B_3(s)] \tag{21}$$

where $L = G_2/G_1$.

Let $f_1(x)$ $(i = 1, 2)$ be the jumps of the displacements across the crack surfaces defined as follows:

$$f_1(x) = u_1(x,0) - u_2(x,0) \tag{22}$$

$$f_2(x) = v_1(x,0) - v_2(x,0) \tag{23}$$

It can be obtained that $f_1(x)$ is an odd function, $f_2(x)$ is an even function.

Applying the Fourier transforms to Eqs. (22) and (23), it can be obtained

$$A_1(s)+A_2(s)-A_3(s) = s\bar{f}_1(s), \quad B_1(s)+B_2(s)-B_3(s) = s\bar{f}_2(s) \tag{24}$$

Here a superposed bar indicates the Fourier transform through the paper. By solving six Eqs. (18)–(21) and (24) with six unknown functions and substituting the solutions into Eqs. (16) and (17) and applying the boundary conditions (6) and (7), it can be obtained

$$\sigma_y^{(2)}(x,0) = \frac{-2G_2}{\pi}\int_0^\infty s[\alpha_1(s)\bar{f}_1(s)+\alpha_2(s)\bar{f}_2(s)]\cos(sx)\,ds = -\sigma_0, \quad 0 \leqslant x \leqslant l \tag{25}$$

$$\tau_{xy}^{(2)}(x,0) = \frac{-2G_2}{\pi}\int_0^\infty s[\alpha_3(s)\bar{f}_1(s)+\alpha_4(s)\bar{f}_2(s)]\sin(sx)\,ds = 0, \quad 0 \leqslant x \leqslant l \tag{26}$$

$$\int_0^\infty \bar{f}_1(s)\sin(sx)\,ds = 0, \qquad \int_0^\infty \bar{f}_2(s)\cos(sx)\,ds = 0, \quad x > l \tag{27}$$

where $\alpha_1(s)$, $\alpha_2(s)$, $\alpha_3(s)$ and $\alpha_4(s)$ are known functions. $\lim_{s\to\infty}\alpha_1(s) = \beta_1$, $\lim_{s\to\infty}\alpha_2(s) = \beta_2$, $\lim_{s\to\infty}\alpha_3(s) = -\beta_2$, $\lim_{s\to\infty}\alpha_4(s) = -\beta_1$. The forms of, $\alpha_1(s)$, $\alpha_2(s)$, $\alpha_3(s)$ and $\alpha_4(s)$ can be seen in Appendix A, respectively. β_1 and β_2 are non-zero constants. It can be written as follows

$$\beta_1 = \frac{1-k_2-L+k_1L}{(k_2+L)(1+k_1L)}, \quad \beta_2 = \frac{1+k_2+L+k_1L}{(k_2+L)(1+k_1L)} \tag{28}$$

When $(\mu_1, \lambda_1, k_1) = (\mu_2, \lambda_2, k_2)$, it can be obtained $\beta_1 = 0$, $\beta_2 = \frac{2}{1+k_1}$. To determine the unknown functions $\bar{f}(s)$ and $\bar{f}_2(s)$, the above two pairs of dual integral Eqs. (25)–(27) must be solved.

4. Solution of the dual integral equations

As in many of previous studies Erdogan and Wu (1993) and Zhang, 1986, in this study too, the problem is solved under the assumption that the effect of the crack surface overlapping very near the crack tips is negligible and there is a sufficiently large component of mode-I loading so that the crack essentially remains open. It can be obtained that the jumps of the displacements across the crack surface are finite, differentiable and continuum functions. Hence, the jumps of the displacements across the crack surface can be expanded by the following series:

$$f_1(x) = \sum_{n=0}^{\infty} a_n P_{2n+1}^{(1/2,1/2)}\left(\frac{x}{l}\right)\left(1-\frac{x^2}{l^2}\right)^{\frac{1}{2}}, \quad \text{for } 0 \leqslant x \leqslant l \tag{29}$$

$$f_2(x) = \sum_{n=0}^{\infty} b_n \, P_{2n}^{(1/2,1/2)}\left(\frac{x}{l}\right)\left(1 - \frac{x^2}{l^2}\right)^{\frac{1}{2}}, \quad \text{for } 0 \leqslant x \leqslant l \tag{30}$$

$$f_1(x) \equiv 0, \quad f_2(x) \equiv 0, \text{ for } x \geqslant l \tag{31}$$

where a_n and b_n are unknown coefficients, $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial (Gradshteyn and Ryzhik, 1980). The Fourier transform of Eqs. (29)–(31) are (Erdelyi, 1954)

$$\bar{f}_1(s) = \sum_{n=0}^{\infty} a_n Q_n \frac{1}{s} J_{2n+2}(sl), \quad Q_n = \sqrt{\pi}(-1)^n \frac{\Gamma(2n+2+\frac{1}{2})}{(2n+1)!} \tag{32}$$

$$\bar{f}_2(s) = \sum_{n=0}^{\infty} b_n R_n \frac{1}{s} J_{2n+1}(sl), \quad R_n = \sqrt{\pi}(-1)^n \frac{\Gamma(2n+1+\frac{1}{2})}{(2n)!} \tag{33}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eqs. (32) and (33) into Eqs. (25)–(27), it can be shown that Eq. (27) are automatically satisfied. After integration with respect to x in $[0, x]$, Eqs. (25) and (26) reduce to

$$\frac{2G_2}{\pi}\left\{\sum_{n=0}^{\infty} a_n Q_n \int_0^{\infty} \frac{\alpha_1(s)}{s} J_{2n+2}(sl)\sin(sx)\,\mathrm{d}s + \sum_{n=0}^{\infty} b_n R_n \int_0^{\infty} \frac{\alpha_2(s)}{s} J_{2n+1}(sl)\sin(sx)\,\mathrm{d}s\right\} = \sigma_0 x, \quad 0 \leqslant x \leqslant l \tag{34}$$

$$\frac{2G_2}{\pi}\left\{\sum_{n=0}^{\infty} a_n Q_n \int_0^{\infty} \frac{\alpha_3(s)}{s} J_{2n+2}(sl)[\cos(sx) - 1]\,\mathrm{d}s + \sum_{n=0}^{\infty} b_n R_n \int_0^{\infty} \frac{\alpha_4(s)}{s} J_{2n+1}(sl)[\cos(sx) - 1]\,\mathrm{d}s\right\} = 0, \quad 0 \leqslant x \leqslant l \tag{35}$$

The semi-infinite integral in Eqs. (34) and (35) can be modified as (for $0 < x < l$):

$$\int_0^{\infty} \frac{\alpha_1(s)}{s} J_{2n+2}(sl)\sin(sx)\,\mathrm{d}s = \frac{\beta_1}{2n+2}\sin\left[(2n+2)\sin^{-1}\left(\frac{x}{l}\right)\right] + \int_0^{\infty} \frac{\alpha_1(s) - \beta_1}{s} J_{2n+2}(sl)\sin(sx)\,\mathrm{d}s \tag{36}$$

$$\int_0^{\infty} \frac{\alpha_2(s)}{s} J_{2n+1}(sl)\sin(sx)\,\mathrm{d}s = \frac{\beta_2}{2n+1}\sin\left[(2n+1)\sin^{-1}\left(\frac{x}{l}\right)\right] + \int_0^{\infty} \frac{\alpha_2(s) - \beta_2}{s} J_{2n+1}(sl)\sin(sx)\,\mathrm{d}s \tag{37}$$

$$\int_0^{\infty} \frac{\alpha_3(s)}{s} J_{2n+2}(sl)\cos(sx)\,\mathrm{d}s = \frac{-\beta_2}{2n+2}\cos\left[(2n+2)\sin^{-1}\left(\frac{x}{l}\right)\right] + \int_0^{\infty} \frac{\alpha_3(s) + \beta_2}{s} J_{2n+2}(sl)\cos(sx)\,\mathrm{d}s \tag{38}$$

$$\int_0^{\infty} \frac{\alpha_4(s)}{s} J_{2n+1}(sl)\cos(sx)\,\mathrm{d}s = \frac{-\beta_1}{2n+1}\cos\left[(2n+1)\sin^{-1}\left(\frac{x}{l}\right)\right] + \int_0^{\infty} \frac{\alpha_4(s) + \beta_1}{s} J_{2n+1}(sl)\cos(sx)\,\mathrm{d}s \tag{39}$$

The semi-infinite integral in Eqs. (36)–(39) can be evaluated directly. Eqs. (34) and (35) can now be solved for the coefficients a_n and b_n by the Schmidt method (Yan, 1967). It can be seen as in Yan (1967).

5. Intensity factors

The coefficients a_n and b_n are known, so that the entire stress field can be obtained. However, in fracture mechanics, it is important to determine stresses $\sigma_y^{(2)}$ and $\tau_{xy}^{(2)}$ in the vicinity of the crack tips. $\sigma_y^{(2)}$ and $\tau_{xy}^{(2)}$ along the crack line can be expressed as:

$$\begin{aligned}\sigma_y^{(2)}(x,0) &= \frac{-2G_2}{\pi}\sum_{n=0}^{\infty}\int_0^{\infty}[a_nQ_n\alpha_1(s)J_{2n+2}(sl)+b_nR_n\alpha_2(s)J_{2n+1}(sl)]\cos(sx)\,\mathrm{d}s \\ &= \frac{-2G_2}{\pi}\sum_{n=0}^{\infty}\int_0^{\infty}[\alpha_nQ_n\{[\alpha_1(s)-\beta_1]+\beta_1\}J_{2n+2}(sl)+b_nR_n\{[\alpha_2(s)-\beta_2]+\beta_2\}J_{2n+1}(sl)]\cos(sx)\,\mathrm{d}s\end{aligned} \tag{40}$$

$$\begin{aligned}\tau_{xy}^{(2)}(x,0) &= \frac{2G_2}{\pi}\sum_{n=0}^{\infty}\int_0^{\infty}[a_nQ_n\alpha_3(s)J_{2n+2}(sl)+b_nR_n\alpha_4(s)J_{2n+1}(sl)]\sin(sx)\,\mathrm{d}s \\ &= \frac{2G_2}{\pi}\sum_{n=0}^{\infty}\int_0^{\infty}[a_nQ_n\{[\alpha_3(s)+\beta_2]-\beta_2\}J_{2n+2}(sl)+b_nR_n\{[\alpha_4(s)+\beta_1]-\beta_1\}J_{2n+1}(sl)]\sin(sx)\,\mathrm{d}s\end{aligned} \tag{41}$$

An examination of Eqs. (40) and (41) shows that, the singular part of the stress field can be obtained from Yan (1967) the relationships as follows:

$$\int_0^{\infty}J_n(sa)\cos(bs)\,\mathrm{d}s = \begin{cases}\frac{\cos[n\sin^{-1}(b/a)]}{\sqrt{a^2-b^2}}, & a>b \\ -\frac{a^n\sin(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n}, & b>a\end{cases}$$

$$\int_0^{\infty}J_n(sa)\sin(bs)\,\mathrm{d}s = \begin{cases}\frac{\sin[n\sin^{-1}(b/a)]}{\sqrt{a^2-b^2}}, & a>b \\ \frac{a^n\cos(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n}, & b>a\end{cases}$$

The singular part of the stress field can be expressed respectively as follows ($l<x$):

$$\sigma = \frac{2G_2\beta_2}{\pi}\sum_{n=0}^{\infty}b_nR_nH_n^{(1)}(x), \quad \tau = \frac{2G_2\beta_2}{\pi}\sum_{n=0}^{\infty}a_nQ_nH_n^{(2)}(x) \tag{42}$$

where

$$H_n^{(1)}(x) = \frac{(-1)^n l^{2n+1}}{\sqrt{x^2-l^2}[x+\sqrt{x^2-l^2}]^{2n+1}}, \quad H_n^{(2)}(x) = \frac{(-1)^n l^{2n+2}}{\sqrt{x^2-l^2[x+\sqrt{x^2-l^2}]^{2n+2}}}$$

The stress intensity factors K_{I} and K_{II} can be written as following:

$$K_{\mathrm{I}} = \lim_{x\to l^+}\sqrt{2\pi(x-l)}\cdot\sigma = \frac{2G_2\beta_2}{\sqrt{l}}\sum_{n=0}^{\infty}b_n\frac{\Gamma\left(2n+1+\frac{1}{2}\right)}{(2n)!} \tag{43}$$

$$K_{\mathrm{II}} = \lim_{x\to l^+}\sqrt{2\pi(x-l)}\cdot\tau = \frac{2G_2\beta_2}{\sqrt{l}}\sum_{n=0}^{\infty}a_n\frac{\Gamma\left(2n+2+\frac{1}{2}\right)}{(2n+1)!} \tag{44}$$

6. Numerical calculations and discussion

As discussed in the works Itou (1986) and Zhou and Wang (2001), it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of infinite series to Eqs. (34) and (35) are retained. The behavior of the sum of the series stays steady with the increasing number of terms in Eqs. (34) and (35). In all computation, the materials are assumed to be the commercially available Iron, Nickel and Aluminum, respectively. The material constants of Iron are $\lambda = 98$ ($\times 10^9$ N/m^2) and $\mu = 77$ ($\times 10^9$ N/m^2), respectively. The material constants of Nickel are $\lambda = 108$ ($\times 10^9$ N/m^2) and $\mu = 66.5$ ($\times 10^9$ N/m^2), respectively. The material constants of Aluminum are $\lambda = 41.4$ ($\times 10^9$ N/m^2) and $\mu = 41.4$ ($\times 10^9$ N/m^2), respectively. The dimensionless stress intensity factors K/σ_0 are calculated numerically. The results of the present paper are shown in Figs. 2–7. From the results, the following observations are very significant:

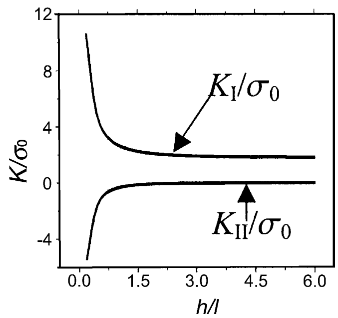

Fig. 2. The stress intensity factor versus h for $l = 1.0$ (the elastic layer and the half plane are the same material, aluminum).

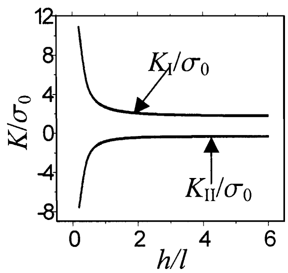

Fig. 3. The stress intensity factor versus h for $l = 1.0$ (material of the elastic layer is aluminum, material of the half plane is nickel).

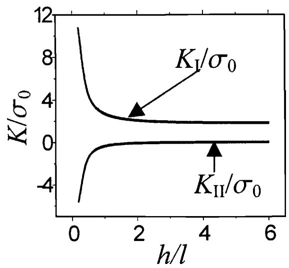

Fig. 4. The stress intensity factor versus h for $l = 1.0$ (material of the elastic layer is nickel, material of the half plane is iron).

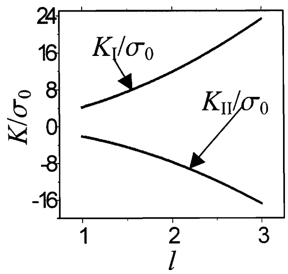

Fig. 5. The stress intensity factor versus l for $h = 0.5$ (material of the elastic layer is aluminum, material of the half plane is nickel).

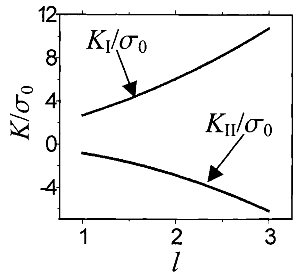

Fig. 6. The stress intensity factor versus l for $h = 1.0$ (material of the elastic layer is aluminum, material of the half plane is nickel).

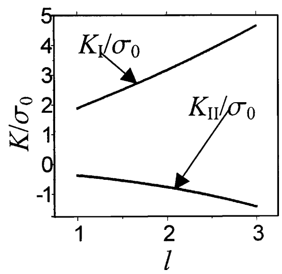

Fig. 7. The stress intensity factor versus l for $h = 3.0$ (material of the elastic layer is aluminum, material of the half plane is nickel).

(i) In the present paper, the unknown variables of dual integral equations are the displacements across the crack surfaces. To solve the dual integral equations, the jumps of the displacements across the crack surfaces are directly expanded in a series of Jacobi polynomials. However, in the previous works England (1965), Erdogan (1965), Rice and Sih (1965), Nisitani et al. (1993), Erdogan and Wu (1993), Dhaliwal et al. (1989), Das and Debnath (2001) and Comninou (1977), the unknown variables of dual integral equations are the dislocation density functions, and need to solve the singular integral equations. This is the major difference. Contrary to the prevision solution of the interface crack, it is found that the stress singularity of the present interface crack solution is of the same nature as that for the ordinary crack in homogeneous materials. The solution of the present paper can be returned to the one of the ordinary crack in the homogeneous materials as shown in Fig. 2.

(ii) The aim of the present paper is to give an approximate approach to resolve the same problem as in Calhoun and Lowengrub (1978). During the solving process, the mathematical difficulties are not met, i.e. the oscillatory stress singularity and the overlapping of the crack surfaces do not meet. The solutions in Dhaliwal et al. (1989), Das and Debnath (2001) and Calhoun and Lowengrub (1978) contain the stress oscillating singularity.
(iii) The normal stress intensity factor K_I/σ_0 increases almost linearly when the length of the crack increases. However, the shear stress intensity factor K_{II}/σ_0 decreases almost linearly when the length of the crack increases as shown in Figs. 5–7. The shear stress intensity factor K_{II}/σ_0 is much smaller than the normal stress intensity factor K_I/σ_0. The shear stress intensity factor K_{II}/σ_0 may be negative for some cases as shown in Figs. 3–7.
(iv) The normal stress intensity factor K_I/σ_0 decreases when the thickness of the elastic layer increases. However, the shear stress intensity factor K_{II}/σ_0 increases when the thickness of the elastic layer increases and it tends to zero for $h/l > 4.0$ as shown in Figs. 3 and 4. The solutions of this paper for $h/l > 4.0$ are approximate to ones of a Griffith crack at the interface of two bonded dissimilar half-planes. It means that the influence of the width of the elastic layer on the results is small for the case $h/l > 4.0$.

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Appendix A

$$m_1(s) = k_2[(1 + k_1L) + (1 - L)e^{-4sh} + (L - 2 - k_1L - 4h^2s^2 + 4h^2Ls^2)e^{-2sh}]$$

$$m_2(s) = L\{(1 + k_1L) + k_1(L - 1)e^{-4sh} + [k_1 + k_1^2L + (L - 1)(1 + 4h^2s^2)]e^{-2sh}\}$$

$$m_3(s) = -(-1 + k_2 + L - k_1L) - (k_2 + L - 1 - k_1L)e^{-4sh}$$

$$m_4(s) = 2(k_2 + L - 1 - k_1L - 2h^2s^2 + 2h^2s^2k_2 + 4h^2s^2L)e^{-2sh}$$

$$m_5(s) = (1 + k_2 + L + k_1L) + (k_2 - L + 1 - k_1L)e^{-4sh}$$

$$m_6(s) = -2(k_2 + 1 + 2h(1 + k_1)Ls + 2h^2s^2 + 2h^2s^2k_2)e^{-2sh}$$

$$m_7(s) = 2(k_2 + 1 - 2h(1 + k_1)Ls + 2h^2s^2 + 2h^2s^2k_2)e^{-2sh}$$

$$\alpha_1(s) = \frac{m_3(s) + m_4(s)}{m_1(s) + m_2(s)}, \quad \alpha_2(s) = \frac{m_5(s) + m_6(s)}{m_1(s) + m_2(s)}, \quad \alpha_3(s) = \frac{-m_5(s) + m_7(s)}{m_1(s) + m_2(s)}, \quad \alpha_4(s) = \frac{-m_3(s) + m_4(s)}{m_1(s) + m_2(s)}$$

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